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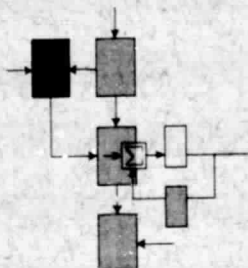
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Final Report for Grant NSG-1112

A Dual-Mode Generalized Likelihood Ratio
Approach to Self-Reorganizing Digital
Flight Control System Design

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Final Report for Grant NSG-1112

A Dual-Mode Generalized Likelihood Ratio Approach
to Self-Reorganizing Digital Flight Control System
Design

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Theoretical Mechanics Branch
Flight Dynamics and Control Division
NASA Langley Research Center
Hampton, Virginia   22365

Attention: Dr. Raymond C. Montgomery

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I. __Introduction__

This report summarizes the results of research supported by NASA Langley Research Center under Grant NSG-1112. Full details of these research efforts have been reported in references [1-6], and in this report we will overview the results, conclusions, and suggestions. The personnel involved in this project have been

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In addition to the results described in the following sections, this project has benefited from interactions with several other research projects. Specifically, the generalized likelihood ratio (GLR) technique has been applied with great success to the detection of arrhythmias in electrocardiograms [7-11], and a study has just been initiated to apply the GLR technique to the detection of traffic incidents on freeways. Also, we have interacted quite closely with the research effort being performed under NASA Langley Contract NAS1-13914, which has been aimed at developing a dual-redundant

sensor failure detection system to be flight tested on the F8 aircraft (see [12-14]). This effort has completed our research effort, which has been aimed at a fundamental, analytical examination of one method for failure detection in dynamic systems.

The Generalized Likelihood Ratio (GLR) technique is a scheme for detecting and identifying abrupt changes in dynamic systems. It attempts to extract all available information about possible abrupt changes in a system. Because it is based on some of the key properties of Kalman filtering, it requires a model of the system being examined. This model allows the GLR technique to make use of the existing analytical redundancy in the system rather than requiring the installation of redundant hardware such as sensors to facilitate fault detection and compensation.

The GLR technique monitors the output of the Kalman filter. If the output is behaving as expected -- i.e., if the filter model accurately predicts the state of the system -- no failure is declared. But if the filter output differs systematically from its predicted behavior a failure is declared and the output is examined closely to determine the time, type, and extent of the failure.

The advantages of GLR include:

1. It can reduce the requirements for multiple redundancy by taking maximal advantage of built-in functional relationships.

2. GLR explicitly searches for the time that the failure occurred, thus allowing it to be sensitive to new data and consequently improving the chances for fast system recovery following detection of a failure.

3. Considerable analysis has been done on GLR behavior. We have found that it is analytically tractable and thus is amenable to detailed design tradeoff studies.

4. A wide range of implementations are available. "Full" GLR looks for any possible failure (of a finite set of types); constrained GLR (CGLR) looks only in certain set of directions; and simplified GLR (SGLR) looks for a certain discrete set of failures. By restricting our attention as in constrained or simplified GLR, we can sometimes improve performance (if only certain directions or values are physically possible) and always reduce the on-line computational load. In addition, simplified GLR is even more analytically tractable than the others.

The goals of our work have been: (1) to develop the analytic tools used to study GLR performance; (2) to test the ability of GLR to distinguish among different failure modes; and (3) to study the sensitivity of GLR to modeling and parameter errors. We have chosen to study these problems in a setting that tests the fundamental limitations of GLR. Specifically, we have applied GLR to the detection of a variety of abrupt changes in a simplified model of the F8 aircraft. In our formulation, we have assumed no hardware redundancy, and hence all failure detection must be solely based on functional redundancy. This clearly accentuates the problems of failure mode distinguishability and GLR sensitivity, but we feel that this approach has yielded a great deal of insight into the characteristics and limitations of GLR.

As mentioned above, a simplified model of the F-8 aircraft has been used as a test bed for our studies. We have used a two dimensional model of the aircraft flying at condition 11, i.e. Mach .6, 20,000 ft., in cumulus clouds. This model is given by

$$x(k+1) = \Phi(k)x(k) + B(k)u(k) + w(k) \tag{1}$$

$$z(k) = H(k)x(k) + J(k)u(k) + v(k) \tag{2}$$

with $x^T = (q,\alpha)$ and q=pitch rate, α=angle of attack. Here, u(k) represents a known control sequence, and w and v are independent zero mean gaussian sequences with covariances Q and R respectively. The Kalman filter is given by

$$\hat{x}(k+1|k) = \Phi(k)\hat{x}(k|k) + B(k)u(k) \tag{3}$$

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)\gamma(k) \tag{4}$$

$$\gamma(k) = z(k) - H(k)\hat{x}(k|k-1) - J(k)u(k) \tag{5}$$

where $\hat{x}(i|j)$ is the estimate of x(i) given the measurements z(1),...,z(j), K(k) is the filter gain, and $\gamma(k)$ is the residual (innovations process) of the filter, which is a white noise process with covariance V(k). The model and Kalman filter parameters appear in Table 1 (here the time step for discretization is 1/32 sec.).

To repeat, in the work reported here, a worst case was considered in that no hardware redundancy was assumed (i.e. we have assumed only one

actuator and sensor of each type). Also, note that B has been left unspe-
cified and was actually taken to be zero, since deterministic inputs do not
affect the residuals.

In Section II, we review and survey the GLR technique. In Section III,
we focus on the issues of detectability and distinguishability. Questions
addressed here include: assuming that the system is known perfectly, can we
detect that a failure has taken place, can we deduce when it happened, and
can we decide which of a restricted class of failures actually occurred?
We briefly survey analytic and simulation results and conclude that detecta-
bility and distinguishability, like controllability and observability, depend
on the structure of the system, but can be studied off-line. In Section IV,
we examine the sensitivity of the GLR technique to modeling errors. We find,
by simulation, that the technique is indeed sensitive to such errors, but
analysis and simulation show that the sensitivity is in certain characteristic
ways, and so compensators may be designed for these difficulties.

II. The Generalized Likelihood Ratio Techniques

The Generalized Likelihood Ratio technique is designed to detect the onset
of abrupt changes in linear systems. Devices based on this method determine
simultaneously whether a change has taken place, the time that the change
occurred, and an estimate of the extent of the change. In the air-craft problem
the abrupt changes considered corresponded to certain types of system failures.
Thus we will refer to all changes as "failures", although their true physical
significance could be quite different (as with an abrupt wind shear).

Four failure modes have been studied intensively:

Table 1

System and Filter Parameters

$$\underline{\Phi} = \begin{bmatrix} 0.93258 & -0.14649 \\ 0.030587 & 0.97193 \end{bmatrix}$$

$$Q^{1/2} = \begin{bmatrix} 0.022596 & 0.0 \\ 0.0043276 & 0.00022603 \end{bmatrix}$$

$$R^{1/2} = \begin{bmatrix} 0.008729834 & 0.0 \\ 0.0 & 0.06 \end{bmatrix}$$

$$H = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 16.154 \end{bmatrix}$$

$$J = 0$$

$$K(k) = \underline{K} = \begin{bmatrix} 7.5351 \times 10^{-1} & 4.6257 \times 10^{-2} \\ 1.3527 \times 10^{-1} & 1.2748 \times 10^{-2} \end{bmatrix}$$

$$cov(x(k|k-1)) = \underline{P}(k|k-1) = \begin{bmatrix} 5.6311 \times 10^{-4} & 1.0891 \times 10^{-4} \\ 1.0891 \times 10^{-4} & 2.2130 \times 10^{-5} \end{bmatrix}$$

$$V(k) = V = \begin{bmatrix} 6.393264579 \times 10^{-4} & 1.759328799 \times 10^{-3} \\ 1.759328799 \times 10^{-3} & 9.374701305 \times 10^{-3} \end{bmatrix}$$

Type 1: state jump

$$x(k+1) = \Phi(k)x(k) + B(k)u(k) + w(k) + \nu\delta_{k+1,\theta} \qquad (6)$$

Type 2: state step

$$x(k+1) = \Phi(k)x(k) + B(k)u(k) + w(k) + \nu\sigma_{k+1,\theta} \qquad (7)$$

Type 3: sensor jump

$$z(k) = H(k)x(k) + J(k)u(k) + v(k) + \nu\delta_{k,\theta} \qquad (8)$$

Type 4: sensor step

$$z(k) = H(k)x(k) + J(k)u(k) + v(k) + \nu\sigma_{k,\theta} \qquad (9)$$

where $\delta_{k,\theta}$ is the Kronecker delta, and $\sigma_{k,\theta}$ is the unit step, which is 1 for $k \geq \theta$ and 0 for $k < \theta$. Here θ has the meaning of the "failure time" and ν is the failure vector of appropriate dimension. We note that the original GLR method devised by Willsky and Jones [15] was developed for type 1 failures.

Other failure types can also be defined as modifications to (1), (2). For example, adding $M u(k)\sigma_{k+1,\theta}$ or $\xi(k)\sigma_{k+1,\theta}$ to (1) defines the hard-over state or increased state noise failures (where M is an appropriate matrix and $\xi(k)$ is a zero mean random process). Similarly adding

$[Lx(k)+Nu(k)]\sigma_{k,\theta}$ or $\xi(k)\sigma_{k,\theta}$ to (2) defines the hardover sensor or increased sensor noise failures. Detectors for these failures have not been studied in detail, although in many cases detectors looking for steps and jumps could be used to detect such changes.[1]

Assume that the Kalman filter (3),(4),(5) has been implemented to estimate the state of the unfailed system (1),(2). When one of the four types of failure occurs, equation (5) produces a residual which is no longer white. In fact,

$$\gamma(k) = \tilde{\gamma}(k) + G_i(k;\theta)\nu \tag{10}$$

where $\tilde{\gamma}$ is the residual in the absence of any failure, and $G_i(k;\theta)$ -- the failure signature matrix -- determines the effect of the type i failure ν that occurred at time θ on the residual at time k. We can establish a set of hypotheses:

H_0 : no failure has occurred

H_i : a failure of type i (ν and θ unknown) has occurred.

Then the generalized likelihood ratio (GLR) is defined by

$$L_i(k) = \frac{p(\gamma(1),\ldots,\gamma(k)|H_i,\ \theta = \hat{\theta}(k),\ \nu = \hat{\nu}(k))}{p(\gamma(1),\ldots,\gamma(k)|H_0)} \tag{11}$$

---

[1]Some work on developing GLR for these hard-over models has been done and is discussed in [1].

where p denotes probability density function; $\hat{\theta}(k)$ and $\hat{v}(k)$ are the maximum likelihood estimates (MLE) of v and θ assuming H_i to be true.

Given H_i is true, the residual is given by (10) for some unknown θ and v. When H_0 is true, the residual is simply $\tilde{\gamma}(k)$.

To choose between H_0 and H_i we use the decision rule:

$$\ell_i(k) \overset{\Delta}{=} 2\ell n(L_i(k)) \underset{H_0}{\overset{H_i}{\gtrless}} \epsilon \tag{12}$$

where ϵ is some predetermined threshold. The estimate $\hat{v}(k)$ can be solved as an explicit function of $\hat{\theta}(k)$:

$$\hat{v}(k) = C_i^{-1}(k;\hat{\theta}(k)) \, d_i(k;\hat{\theta}(k)) \tag{13}$$

where $C_i(k;\theta)$ is the matrix

$$C_i(k;\theta) = \sum_{m=\hat{\theta}}^{k} G_i^T(m;\theta) v^{-1}(m) G_i(m;\theta) \tag{14}$$

and $d_i(k;\hat{\theta})$ is a linear combination of the residuals:

$$d_i(k;\theta) = \sum_{m=\hat{\theta}}^{k} G_i^T(m;\hat{\theta}) v^{-1}(m) \gamma(m) \tag{15}$$

Then $\hat{\theta}(k)$ is the value of $\theta \leq k$ that maximizes $\ell_i(k;\theta)$:

$$\ell_i(k;\theta) = d_i^T(k;\theta) C_i^{-1}(k;\theta) d_i(k;\theta) \qquad (16)$$

Therefore, the (full) GLR system will declare a type i failure $\hat{\nu}$ occurring at $\hat{\theta}$ if $\ell_i(k;\hat{\theta}) > \epsilon$ and $\ell_i(k;\hat{\theta}) \geq \ell_j(k;\theta)$ for $1 \leq \theta \leq k$ and $j=1,2,3,4$. As time progresses, the number of possible values of $\hat{\theta}$ increases. Hence, the implementation of this scheme involves a growing bank of filters to implement (15) for each θ.

A number of simplifications of the approach have been suggested by Willsky and Jones [15] such as the finite window assumption where $\hat{\theta}$ is restricted to a range, $k-M \leq \hat{\theta} \leq k-N$. The physical assumptions made here are: 1) no decision can be made with less than N observations (an observability constraint), 2) failures before time k-M should have been detected at an earlier time and compensated for already. The resulting "sliding window" reduces the computational burden imposed by the growing bank of filters. When the system under consideration is time invariant and the associated Kalman-Bucy filter (KBF) has reached a steady state, the G and C matrices become dependent on k-θ only. Thus, these matrices may be computed once and stored, greatly simplifying the required calculations. To reduce required calculations even further, G and C can be approximated by polynomials, or a sequence of ramp and step functions. Also, (15) may be viewed as a convolution (it is a correlation calculation, as in a matched filter) and may then be calculated very rapidly using the Fast Fourier Transform.

Another simplification is constrained GLR (CGLR) which involves the assumption that $\nu = \alpha f_j$ where α is a scalar and f_j is one of a finite set

of directions. The CGLR detector (for a single failure type) takes the form:

$$\hat{\alpha}(k) = \frac{b(k; \hat{\theta}(k), \hat{j}(k))}{a(k; \hat{\theta}(k), \hat{j}(k))} \tag{17}$$

where $\hat{\theta}(k)$ and $\hat{j}(k)$ are the quantities that maximize

$$\ell(k; \theta, j) = \frac{b^2(k; \theta, j)}{a(k; \theta, j)} \tag{18}$$

$$a(k; \theta, j) = f_j' C(k; \theta) f_j \quad , \quad b(k; \theta, j) = f_j' d(k, \theta) \tag{19}$$

$$\ell(k; \hat{\theta}(k), \hat{j}(k) \underset{H_0}{\overset{H_i}{\gtrless}} \epsilon \tag{20}$$

Clearly we can use CGLR when we have several failure types.

If ν is further restricted to be some constant ν_0, one has simplified GLR (SGLR). We note that SGLR does not require maximization over ν and hence $\ell(1; \hat{\theta})$ becomes

$$\ell(k; \theta) = \sum_{m=1}^{k} [2\gamma(m) - G(m; \theta)\nu_0]' V^{-1}(m) G(m; \theta)\nu_0 \tag{21}$$

Here we simply choose $\hat{\theta}(k)$ to maximize (21) and declare a failure if $\ell(k; \theta)$ is over a threshold. Note that (21) is linear in γ and therefore $\ell(k; \theta)$ is a gaussian random variable. This makes SGLR more amenable to

analysis than full or constrained GLR.

Both CGLR and SGLR require less computation than full GLR. They are directionalized, i.e., most sensitive to certain directions. This can be of great practical value where only certain kinds of failures are physically meaningful. Such knowledge can greatly decrease sensitivity problems and can vastly improve our ability to distinguish among the various failure types.

It is clear that the GLR method offers a range of implementations from the point of view of computational complexity. In order to develop a useful detector design methodology, one must study much more carefully the properties of the GLR method and the tradeoffs inv·lved in the design. Our research has intended to provide some guidelines for the use of the GLR technique. Our aim provided some guidelines for the use of the GLR technique. Our aim has been to develop a framework in which one can systematically study the various tradeoffs that arise in detecting failures by this approach.

III. Detectability and Distinguishability

In this section, we discuss certain characteristics of GLR when the system parameters - except for the failure terms - are known perfectly. Here, the major issues are detectability (can the system find that a failure has occurred without great risk of false alarms, and without great delay?) and distinguishability (can the system properly identify the type of failure, the failure vectors, and (perhaps) the failure time?).

A. Performance Probabilities

The probability of correct detection (P_D) is the probability that ℓ_i is above the threshold ε at time k if a failure of type i (with failure

vector ν) occurred at time $\theta < k$. The probability of <u>false alarm</u> (P_F) is the probability that $\ell_i(k;\theta)$ is above ϵ when no failure actually occurred. The cumulative distribution function P_{TD} is the probability that the <u>detection time</u> (i.e. the first time $k \geq \theta$ that $\ell_i(k;\theta) > \epsilon$ when we have a type i failure at time θ with vector ν) k is less than a given time T. The probability of choosing the wrong time P_{WT} is the probability of $\ell_i(k,\theta)$ being above the threshold when the failure actually occurred at $\theta_t \neq \theta$. The <u>cross detection</u> probability P_{CD} is the probability of ℓ_i being above the threshold when the failure was actually of type $j \neq i$. P_{CW} is the probability that <u>both</u> these events have taken place, and is the <u>wrong time-cross detection</u> probability.

$$P_D(k,i,\theta,\nu) \overset{\Delta}{=} \text{Prob } (\ell_i(k;\theta) > \epsilon \,|\, H_i,\theta,\nu) \tag{22}$$

$$P_F(k,i,\theta) \overset{\Delta}{=} \text{Prob } (\ell_i(k;\theta) > \epsilon \,|\, H_0) \tag{23}$$

$$P_{TD}(T,i,\theta,\nu) \overset{\Delta}{=} \text{Prob } (\ell_i(k;\theta) > \epsilon \text{ for some}$$
$$\theta \leq k \leq T \,|\, H_i,\theta,\nu) \tag{24}$$

$$P_{WT}(k,i,\theta_t,\nu,\theta) \overset{\Delta}{=} \text{Prob } (\ell_i(k,\theta) > \epsilon \,|\, H_i,\theta_+,\nu) \tag{25}$$

$$P_{CD}(k,i,j,\theta,\nu) \overset{\Delta}{=} \text{Prob } (\ell_i(k,\theta) > \epsilon \,|\, H_j,\theta,\nu) \tag{26}$$

$$P_{CW}(k,i,j,\theta,\theta_t,\nu) = \text{Prob } (\ell_i(k,\theta) > \epsilon \,|\, H_j,\theta_t,\nu) \tag{27}$$

To calculate these probabilities, we closely examine the GLR equations. It can be shown [3] that $\ell_i(k,\theta)$ is a noncentral χ^2 random variable with n degrees of freedom[1] when the true failure is of type j, magnitude ν, time θ_t. The non-centrality parameter is given by

$$\delta^2 = \nu^T C_{i|j}^T(k;\theta|\theta_t) C_{i|i}^{-1}(k;\theta|\theta) C_{i|j}(k;\theta|\theta_t)\nu \tag{28}$$

where

$$C_{i|j}(k;\theta|\theta_t) = \sum_{m=\max(\theta,\theta_t)}^{k} G_i^T(m;\theta) V^{-1}(m) G_j(m;\theta_t) \tag{29}$$

Note that the expected value of $\ell(k;\theta)$ is $n+\delta^2$.

Figure 1 shows δ^2 as a function of $k-\theta$ where $\epsilon=14$ (so $P_F=.000912$), $\nu^T=(.0226,0)$ in the curves marked q, and $\nu^T=(0,.00043)$ in the curves marked α. These curves are for state step failures in the F-8 model described in Section I. Figure 2 plots $\delta_{2/2}^2(k,\theta)$ and $\delta_{4/2}^2(k,\theta)$ for wrong time and correct and (wrong time) cross detection. Here a state step failure with vector $\nu^T=(.00226,0)$ occurred at $\theta=k-30$.

This plot displays some of the cross detection and wrong time difficulties that may arise. Note that $\delta_{4/2}^2(k,\theta) > \delta_{2/2}^2(k,\theta)$ (i.e., the wrong detector is more likely to trigger than the correct one) for all θ except the correct failure time. Furthermore, the curvature of $\delta_{2/2}^2(k,\theta)$ is very shallow near $\theta=k-30$. Thus, the system may have great difficulty in choosing the correct θ.
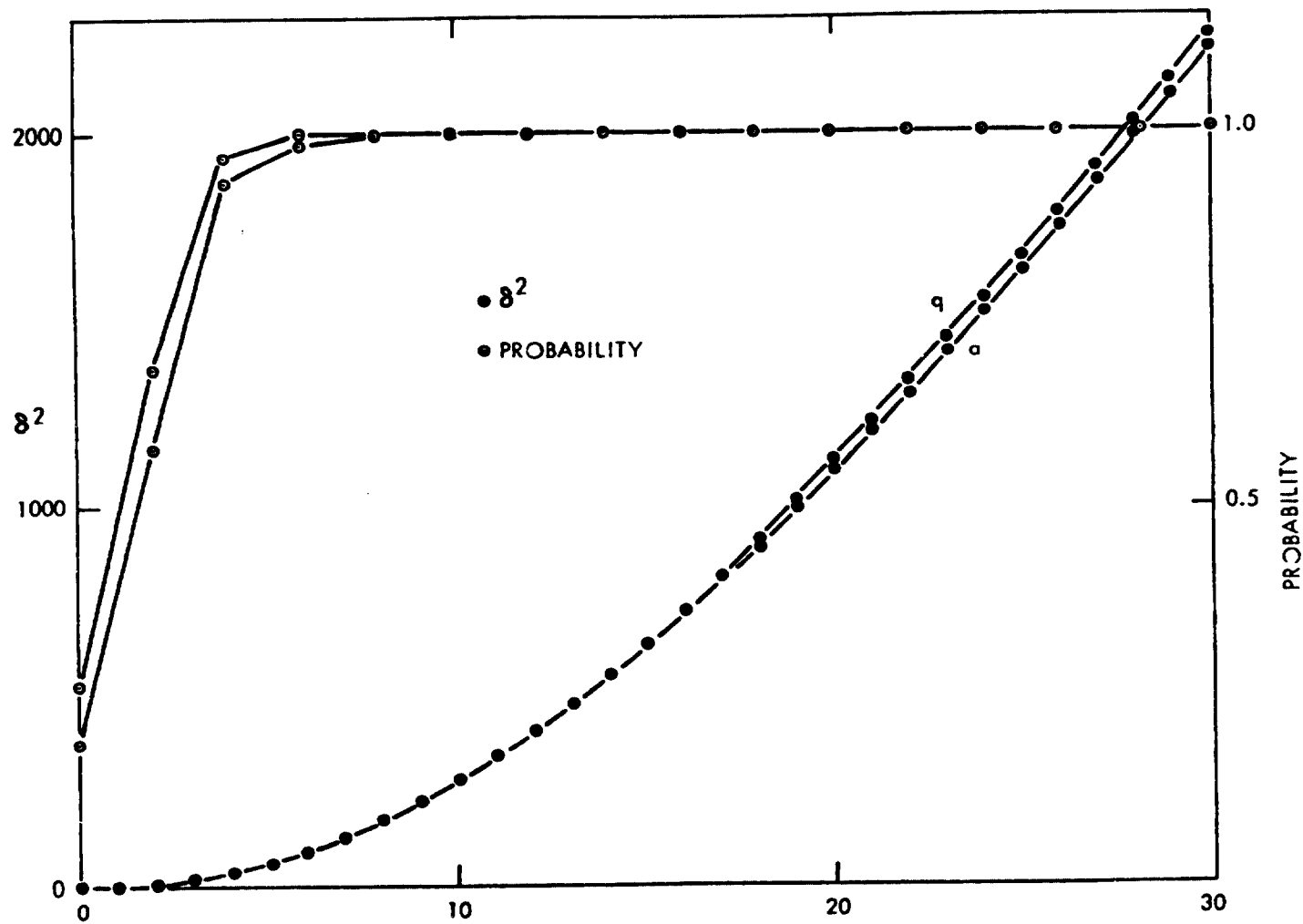
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[1] The dimension of ν is n.

Fig. 1   1σ State Step δ^2 and P_D with $\epsilon=14$ ($P_F=0.000912$)
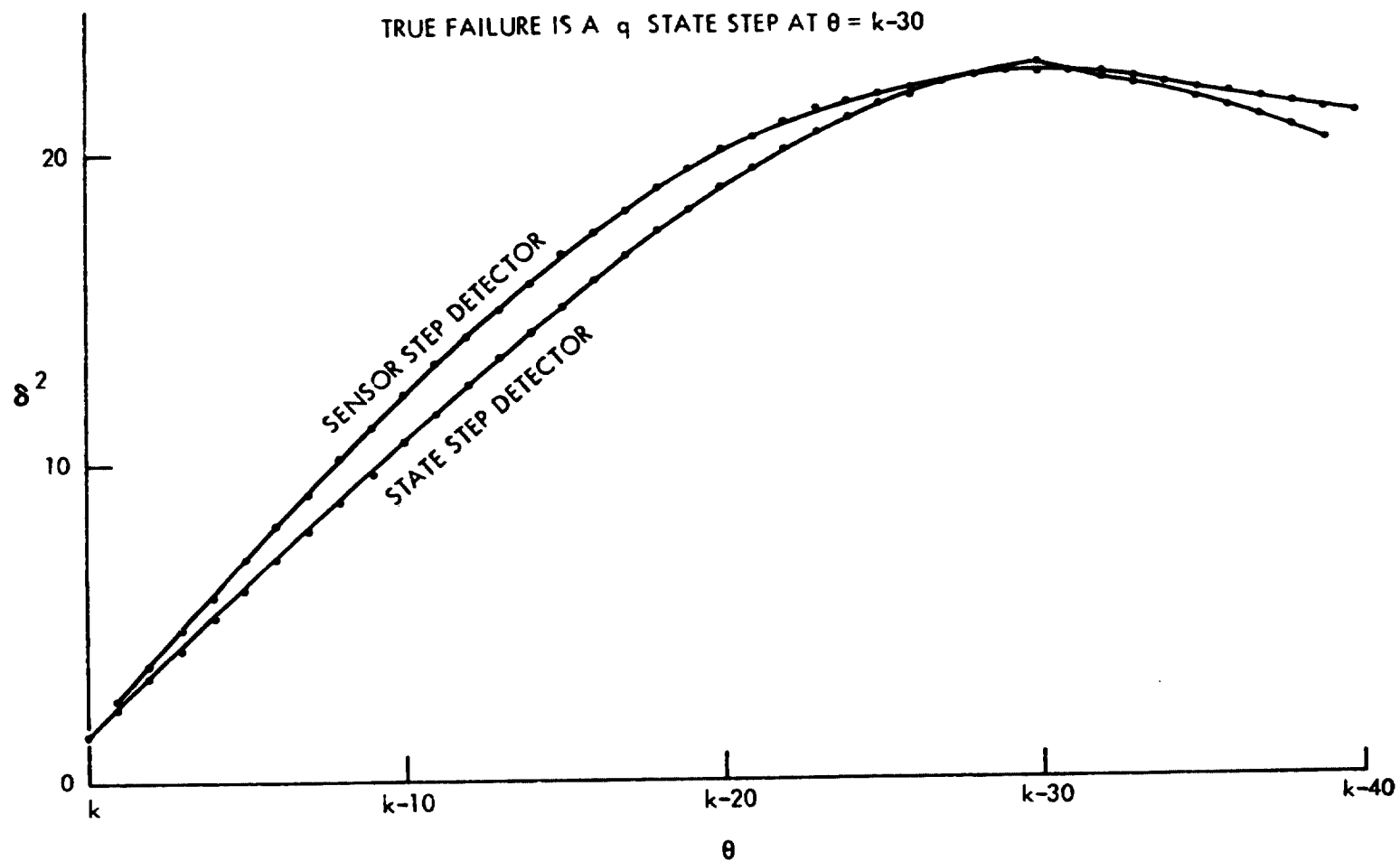
Fig. 2   Wrong Time Cross Detection Noncentrality Parameter

Figure 3 displays $P_D(\delta^2, \varepsilon)$ for various values of δ^2 and $P_F(\varepsilon)$ as functions of ε, the threshold. Note that these curves depend only on the non-central χ^2 distribution with 2 degrees of freedom and not at all on any other information about the system or its failures. All other such information is summarized in δ^2. Note that for a given P_F, ε is determined, and that among P_F, P_D, and δ^2, any two determine the third. In general, to increase the spread between P_F and P_D, δ^2 must be increased.

B. The Information Matrix

The C matrices are indicators of sensitivity and distinguishability of failures in the GLR technique. In particular, the inverse of

$C_i(k;\theta) = C_{i|i}(k;\theta|\theta)$ is equal to the covaraince of the estimate $\hat{\nu}(k)$

(assuming we correctly determine θ). Under conditions of correct detection at the correct time,

$$\delta^2 = \nu^T C_i(k;\theta)\nu \tag{30}$$

Thus C indicates directions (in failure space) of greater or lesser sensitivity to failures, and thus gives us a measure of signal-to-noise ratio. The greater δ^2, the larger the mean of $\ell(k,\theta)$ and the greater the likelihood of exceeding the threshold.

Note that (13) requires the inversion of $C_i(k;\theta)$. If this matrix is not invertible, failures in some direction cannot be detected. Thus, the requirement for universal <u>detectability</u> is that C is invertible. This condition is analogous
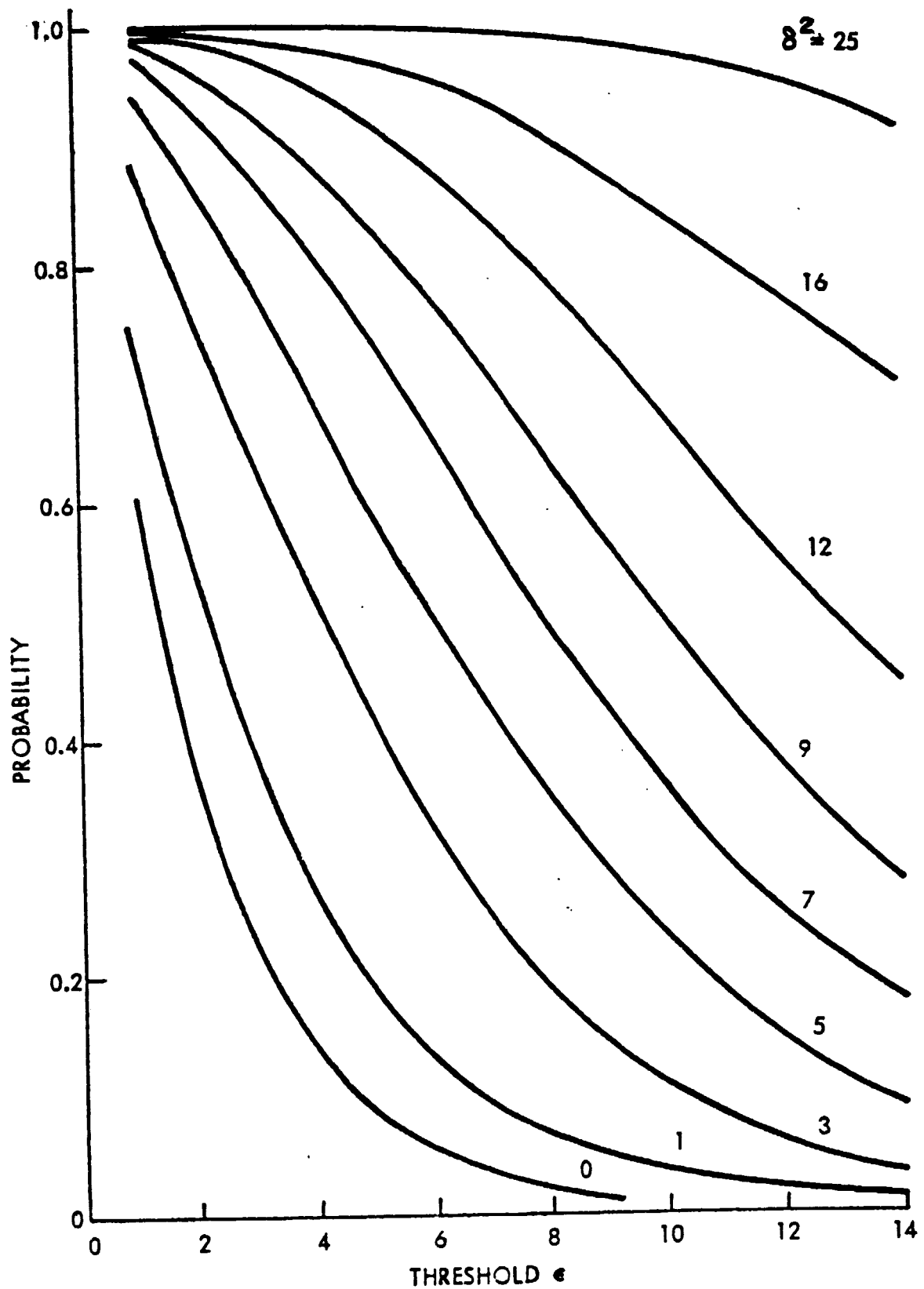
FIG. 3 $P_D(\delta^2, \epsilon)$ FOR A VARIABLE WITH 2 DEGREE OF FREEDOM

to observability in linear systems and can be verified in advance. We note that sensor failures are <u>always</u> observable, while the observability of dynamics failures is directly related to the observability of the original system [3].

Equation (28), in general, shows how errors of failure type or failure time affect the detector. If $C_{i|j}(k;\theta|\theta)\nu$ is large for some ν, then the system is likely to erroneously declare a failure type i at time θ, when, in fact, a failure of type j occurred at time θ_t.

From (29), $\mu^T C_{i|j}(k;\theta|\theta)\nu$ can be thought of as an inner product between $G_i(\cdot;\theta)\mu$ and $G_i(\cdot;\theta_t)\nu$, i.e. the inner product of the signature of a type i failure at time θ and a type j failure at time θ_t. When this is large, the failures are likely to be nearly indistinguishable, but when they are nearly orthogonal, we expect no difficulty in distinguishing between these failure modes. We note that the analysis for CGLR is quite similar, but in that case the directions μ and ν are restricted to fixed finite sets (although their magnitudes are free). The analysis for SGLR is even simpler.

C. Behavior of the Log Likelihood Ratios

One reason that analysis of GLR performance is nontrivial is that the log likelihood ratios, in full or constrained GLR, are noncentral χ^2 random variables. However, in simplified GLR, $\ell_i(k;\theta)$ is given by (21) and is a gaussian random variable. The mean of $\ell_i(k;\theta)$ and the covariance of $\ell_i(k_1,\theta_1)$ and $\ell_i(k_2,\theta_2)$ can be easily calculated [6].

These quantities are important because the probabilities calculated in Section II.A are all static (except for the distribution P_{TD}) in the they consider the probability of $\ell(k,\theta) > \epsilon$ for a given k and θ. By considering ℓ as a (correlated) random process, we can calculate the probability of events such as $\{\ell(k,\theta) > \epsilon, k=k_1,\ldots,k_2\}$. Thus we can create more sophisticated detection rules which look at sequences of ratios and which can decrease the false alarm rate, cross detection difficulties, etc. (See also the sensitivity problem discussed in the next section).

D. Simulation Experience

Numerous computer simulations have been performed using the F-8 model described in Section I. It was found that for any of the four failure types described in Section II, detection occurs within two seconds if the failure size is on the order of the standard deviation of the noise. Thus detectability is not a problem.

Distinguishability, however, can lead to difficulties. Figure 4 shows a plot of $\ell(k,\theta)$ for a simulation run in which a step failure in the state occurred (type 2). The detector was based on that failure. Figure 5 shows a plot of $\ell(k,\theta)$ from a run in which a step failure in the q-sensor (type 4) occurred but where the detector was looking for a step failure in the state (type 2). Notice how similar these plots are. This implies that it would be very difficult to distinguish between these failures by looking at $\ell_i(k,\theta)$.

The reason for this is that full GLR allows the estimate $\hat{v}(k)$ to be any point in space. In this case $\hat{v}_{2/2}(k)$ and $\hat{v}_{2/4}(k)$ have been found such that
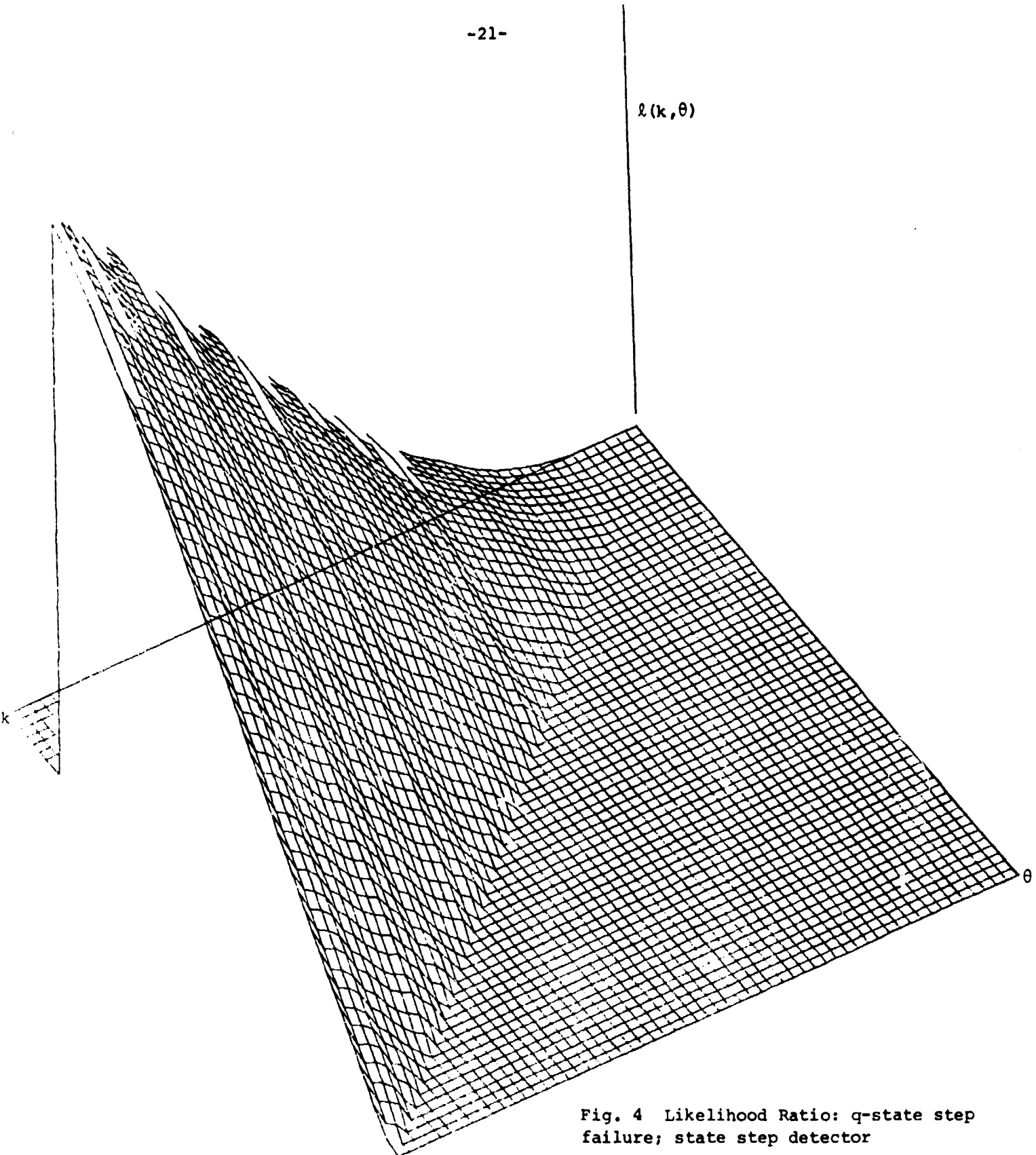
$\ell(k,\theta)$

Fig. 4 Likelihood Ratio: q-state step
failure; state step detector

$\ell(k,\theta)$
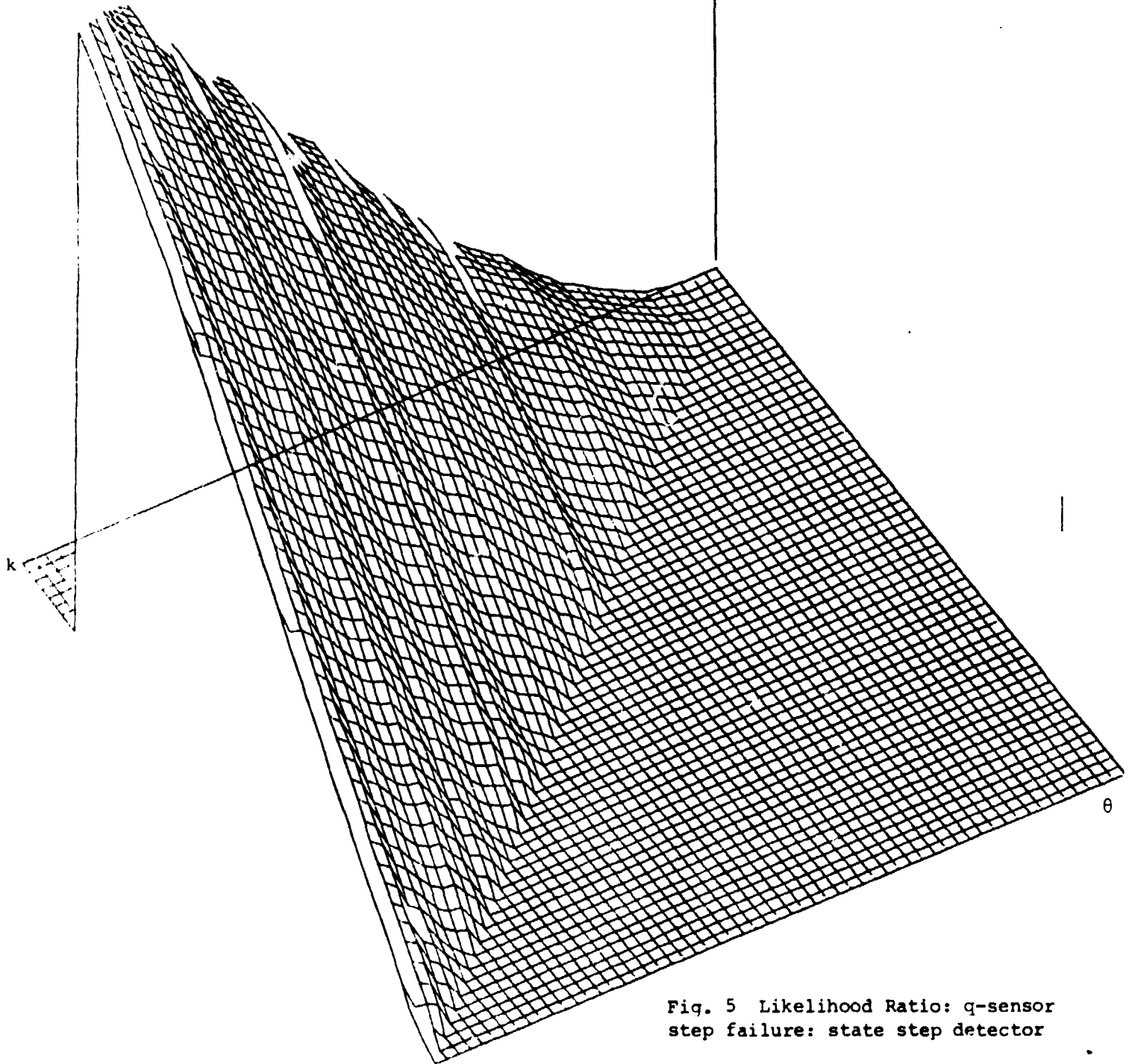

Fig. 5 Likelihood Ratio: q-sensor
step failure: state step detector

$$\ell_2(k,\theta) = \ell_{2/2}(k,\theta) = \hat{v}_2^T(k)C_2(k;\theta|\theta_t)\hat{v}_2^T(k)$$

and

$$\ell_{2/4}(k,\theta) = \hat{v}_{2/4}^T(k)C_{2/4}^T(k;\theta|\theta_t)C_{4/4}^{-1}(k;\theta|\theta_t)C_{2/4}(k;\theta|\theta_t)\hat{v}_{2/4}(k)$$

take on approximately equal values for all k,θ.

There are two ways out of this difficulty. One is to use constrained or simplified GLR. It is likely that the wrong estimate, $\hat{v}_{2/4}(k)$ in this case, is not physically meaningful. If we had restricted our attention only to meaningful failure directions and, perhaps, magnitudes, much ambiguity would have been eliminated. Another point to note is that the wrong estimates of v and θ are likely to change much more with k than the correct estimates, and t is can help in choosing the correct failure type. Research investigating performance improvements using these ideas is continuing.

Finally, it should be noted that the magnitude of the state failure that leads to Figure 4 is on the same order as the noise in the dynamics. However, to get Figure 5 with likelihood ratios approximately the same as in Figure 4 requires a q-sensor failure on the order of more than 10 times the magnitude of the noise in the sensor. Hence, if we used a threshold to detect step failures of (say) 10σ, it would take a 100σ q-sensor failure to confuse us. Issues such as this, involving notions of signal to noise ratios, can be analyzed by detailed examination of the $C_{i/j}$, together with knowledge concerning reasonable expected magnitudes for the various failure modes.

It should be pointed out that we have demonstrated a particularly nasty example. Figure 6 displays $\ell(k,\theta)$ for a run in which a step failure in the

α-sensor (type 4) took place, and where the detector, again, was designed for a step failure in the state (type 2). The marked difference between Figure 6 and Figure 4 indicates that while there may be distinguishability difficulties between state step and q-sensor step failures, there are no such difficulties between state step and α-sensor step failures in the F-8 (if one can utilize the full information in $\ell(k,\theta)$ as a function of both k and θ).

E. Conclusions

Using a combination of analytic and simulation approaches, the GLR technique has been extensively studied under the assumption that the system is perfectly modeled. The four failure types investigated are satisfactorily detectable but can lead to problems in distinguishability. The key issues in this problem have been isolated. Further research should resolve this difficulty.

IV. Sensitivity to Modeling Errors

In designing GLR failure detectors, it is necessary to hypothesize a certain unfailed system (1),(2). In this section, we examine the behavior of this technique when the model parameters used in calculating the Kalman filter gains and the detector matrices $C_i(k,\theta)$, $G_i(k,\theta)$ do not correspond with those of the true system. Not surprisingly, performance is somewhat degraded. However, in view of the large modeling errors assumed, and in the manner in which performance is affected, there is hope that such errors can be compensated for. In any event, one can use these results to determine the size of failures that can be detected. That is, if we view these
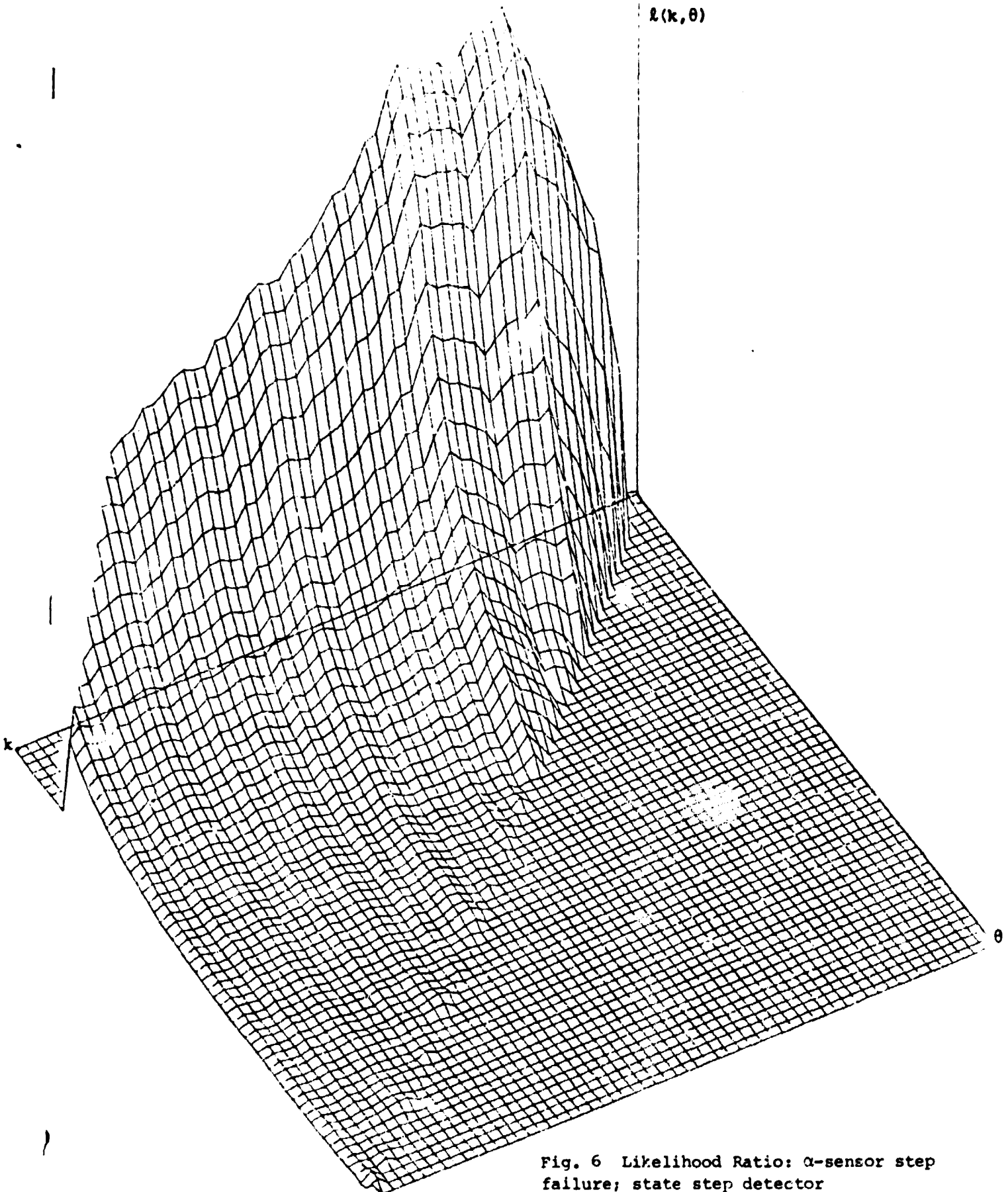
$\ell(k,\theta)$

k

θ

Fig. 6 Likelihood Ratio: α-sensor step
failure; state step detector

modeling errors as unavoidable, we may decide to set detector thresholds
higher to avoid false alarms. This in turn increases the magnitude of
each failure type that is needed to achieve a given probability of detection.

A. Description of Mismatch

In the mismatches we consider, the true system is the simplified model
of the longitudinal dynamics of the F-8 at either flight condition 10 or 12,
i.e., at Mach .4 or .8, 20,000ft., with cumulus clouds. The system on which
the Kalman filter and detector were based was at condition 11, i.e. Mach .6
with the same altitude and weather conditions. These systems differ in two
important ways. Although in all systems the sensor H matrix is diagonal,
the $H_{\alpha\alpha}$ term, which depends on the dynamic pressure, changes by a factor of
2 from condition 10 to 11 and again from 11 to 12. In addition, while the
aircraft oscillates under all three conditions, its period drops sharply from
condition 10 to 11 and from 11 to 12.

These changes affect the behavior of the quantities of interest in the
GLR technique: γ, ℓ, $\hat{\theta}$, $\hat{\upsilon}$. However, they affect them in very specific ways,
and by studying this, some compensation is possible.

B. Behavior of GLR Random Variable

Because the Kalman filter no longer matches the system, it is no longer
true that the unfailed residuals form a white noise process. However, it can
easily be shown that $\gamma(k)$ is the output of a process which oscillates with the
same period as that of the true system, i.e. flight condition 10 or 12 here.

Because $\gamma(k)$ is oscillatory, and because ℓ is a quadratic function of γ, $\ell(k,\theta)$ is also oscillatory: both as a function of k for fixed θ and as a function of θ for fixed k. Figure 7 shows $\ell_2(k,\theta)$ (i.e., the likelihood ratios of the full GLR state step detector) when the true system is at condition 12, and where no failure has taken place. The maximum value of ℓ here is over 1000, so that this will surely precipitate a false alarm. It should be pointed out that under mismatch conditions (but <u>not</u> under matched conditions) the value of the system vector is important. Here, at $k=0$, $q=0$ and $\alpha=5^0$, which is quite large. Under more quiescent conditions, the mismatch will be far less apparent.

This shows that we cannot simply as if ℓ is greater than a threshold, but we must look also at the behavior of ℓ and other quantities. For instance, as long as the first peak of ℓ is the highest ℓ in the window (Figure 6), $\hat{\theta}(k)$ will correspond to that peak. If the first peak leaves and the second peak enters, then at some k there will be an abrupt shift in $\hat{\theta}(k)$. This oscillatory behavior of ℓ and the shift in $\hat{\theta}$ are characteristic of mismatches and are important indicators. In addition, $\hat{\nu}(k)$, which may be time-varying as discussed earlier, here jumps abruptly with $\hat{\theta}(k)$.

Figures 8 and 9 show the behavior of $\ell_2(k,\theta)$ when a state step (type 2) failure and a sensor step (type 4) failure take place. In both cases ℓ is well over the threshold discussed in Section III, but the maximum value of ℓ in Figure 8 is slightly less than of Figure 6, while that in Figure 8 is nearly 4000. (However, it is <u>not</u> always true that the correct detector has
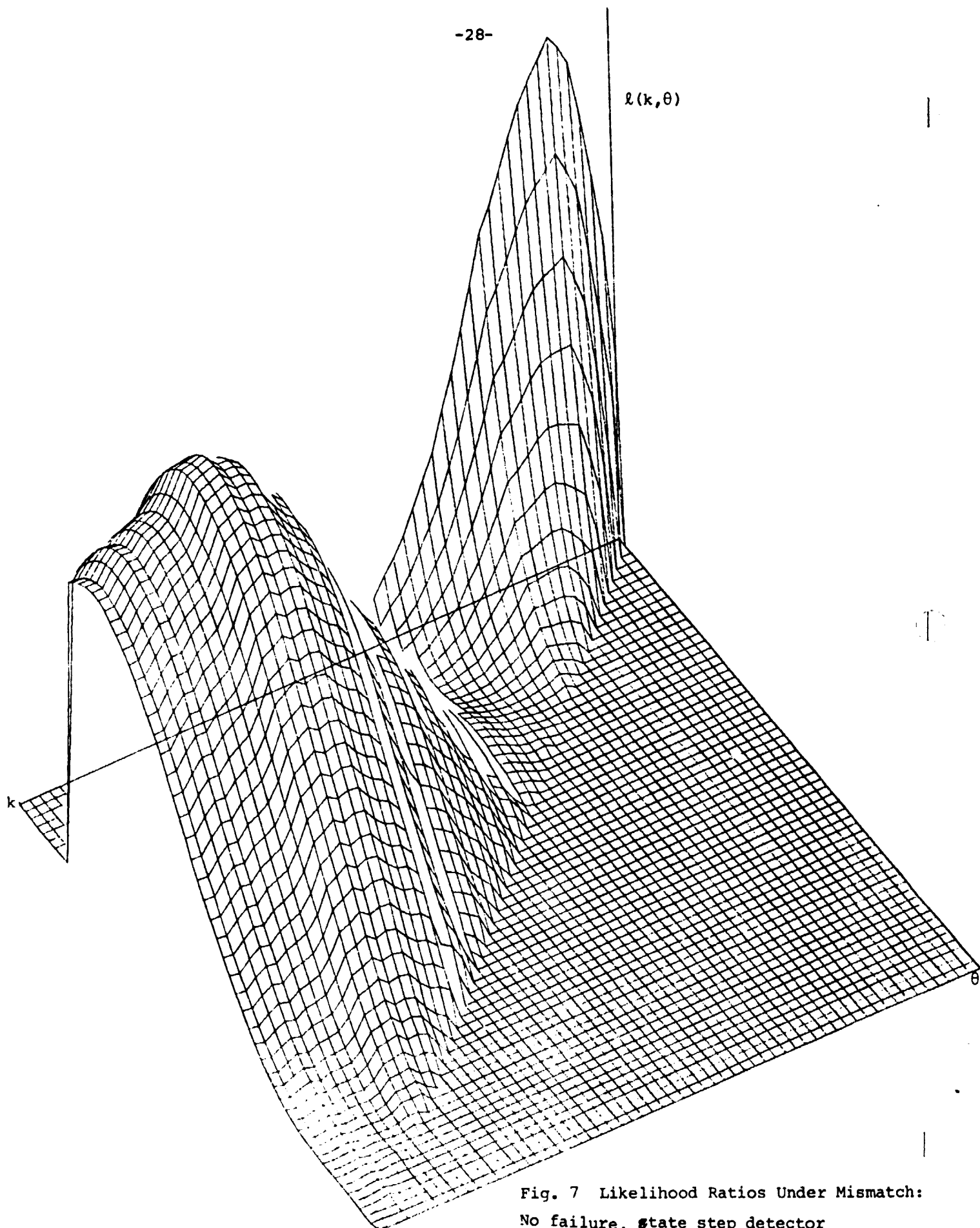
$\ell(k,\theta)$

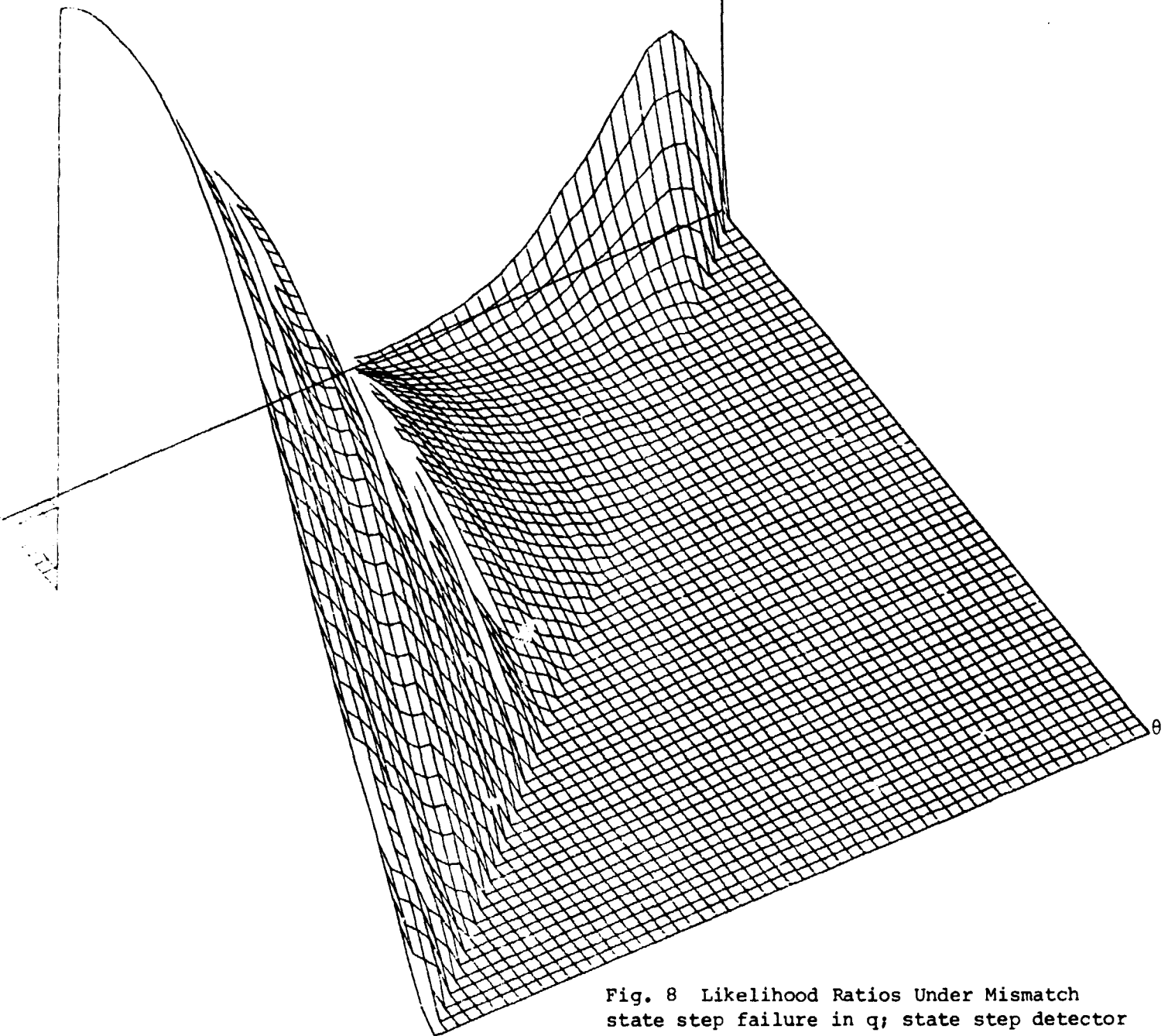Fig. 7 Likelihood Ratios Under Mismatch:
No failure, state step detector

$\ell(k,\theta)$

θ

Fig. 8 Likelihood Ratios Under Mismatch
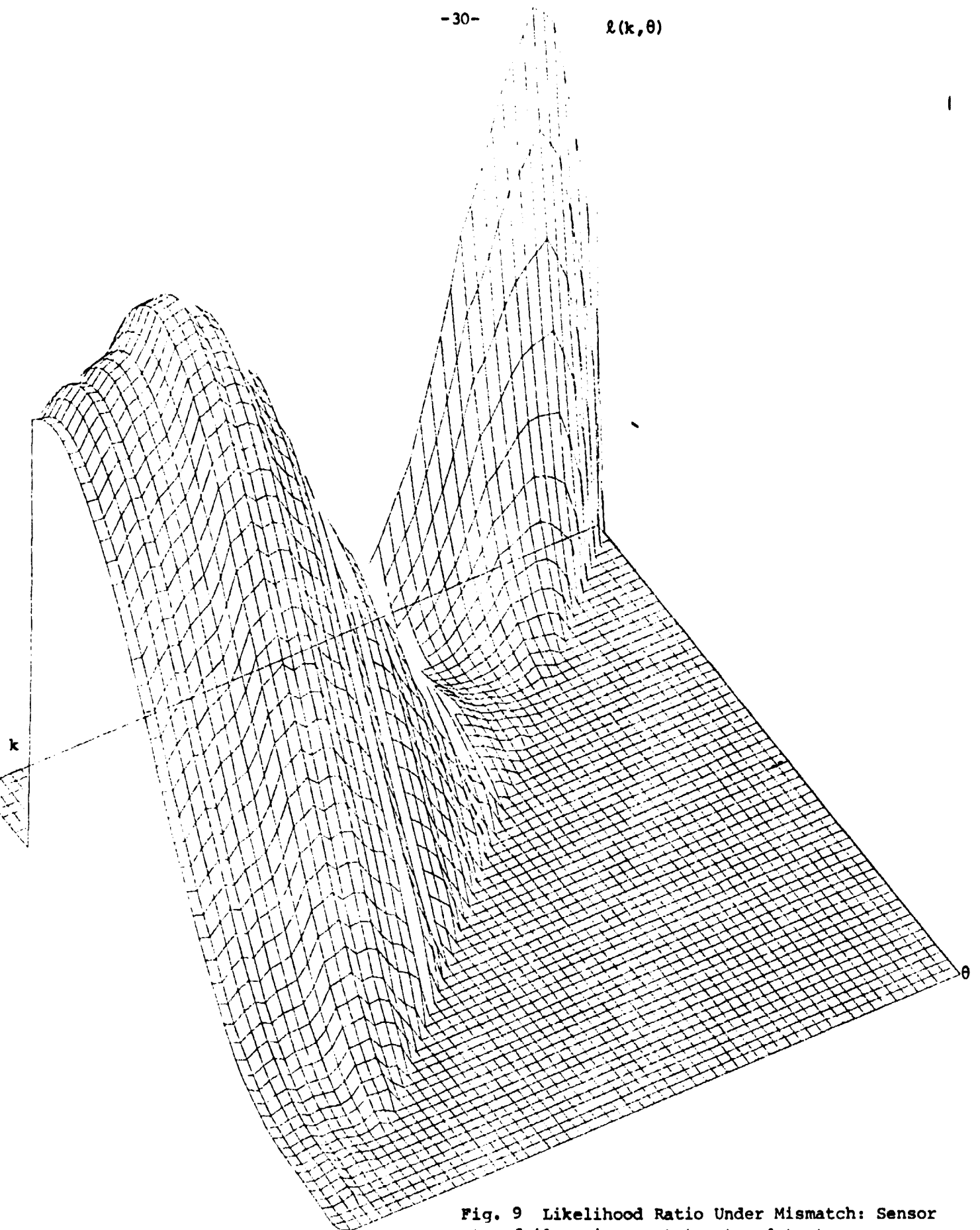state step failure in q; state step detector

$\ell(k,\theta)$

k

θ

Fig. 9 Likelihood Ratio Under Mismatch: Sensor
step failure in q; state step detector

higher $\ell(k,\theta)$ than the wrong detectors, especially under mismatch conditions).

Thus we see that although the mismatch is severe, the detector for state steps yields a far larger likelihood ratio under failure conditions (Figure 8) than when here is no failure (Figure 7). This indicates that by simply raising detector threshold, we can minimize the effect of system mismatch. Of course, one would haope to utilize more information about the shape of the $\ell(k,\theta)$ contours in these figures to do even more. This awaits further work.

C. Compensation for Mismatch

It was pointed out that one of the major effects of mismatch is the change in the $H_{\alpha\alpha}$ which is related to dynamic pressure. We have considered the possibility that we may be able to measure dynamic pressure directly so thatwe can effectively correct the mismatch in the α sensor. In Figure 10 $\ell(k,\theta)$ is displayed for a state step detector constructed in this way.

The maximum value of ℓ is now only 25. Therefore, we can expect far fewer false alarms. Further studies are required to determine if this improves cross detection and mismatch performance.

It would appear that some mismatch difficulties can be resolved by designing GLR to be operated in conjunction with an adaptive estimation control system such as the multiple model method [16]. The latter would be responsible for detecting shifts in parameters (possibly including hard-over failures), and the former would search for abrupt additive changes.

$\ell(k,\theta)$
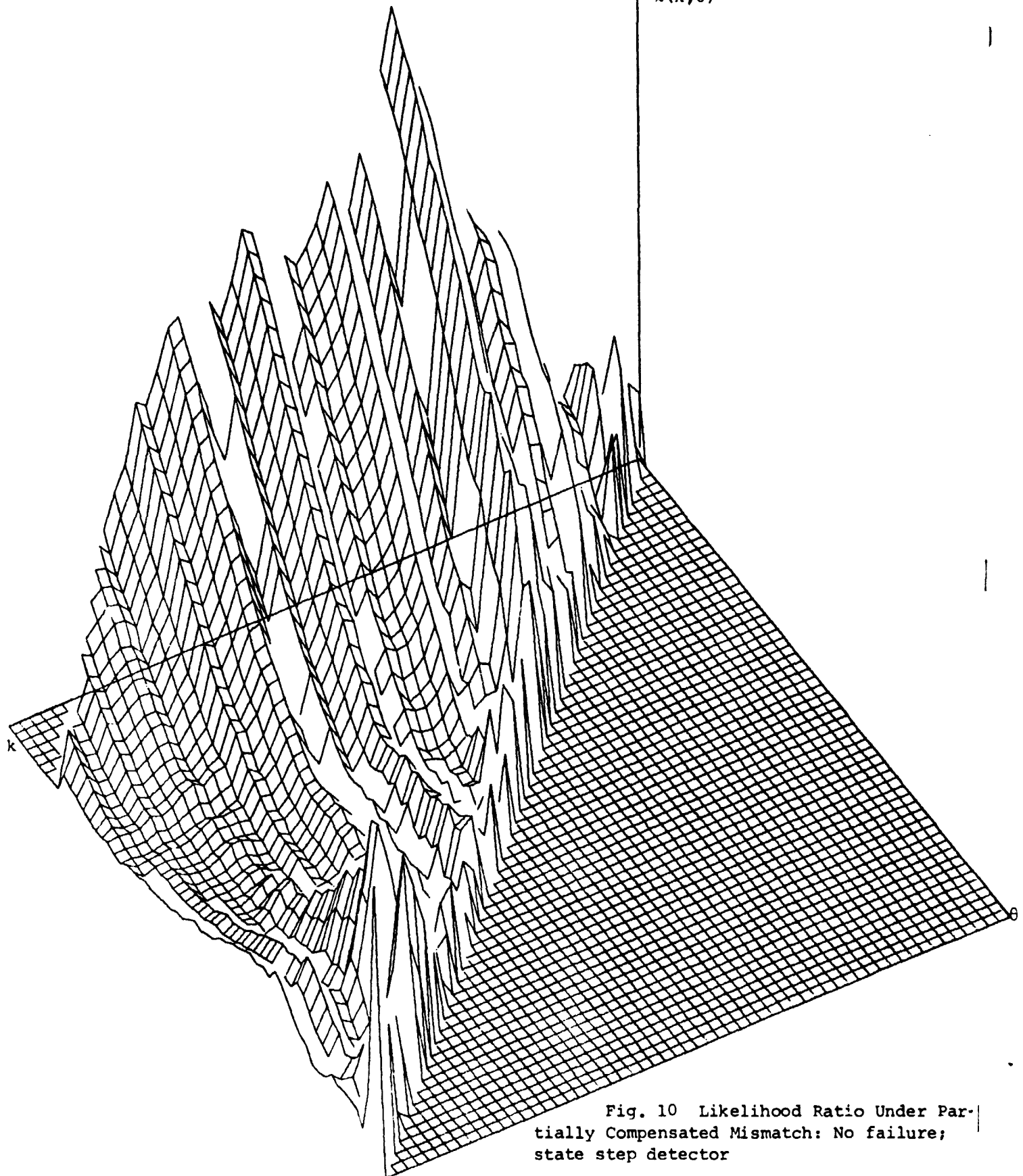
Fig. 10 Likelihood Ratio Under Partially Compensated Mismatch: No failure; state step detector

V. Conclusions and Suggestions

The Generalized Likelihood Ratio (GLR) method for failure detection has been intensively studied. Many of the basic issues such as detectability, distinguishability, and parameter sensitivity have been identified and analyzed. The F-8 model has proved to be a useful setting in which to formulate problems and test ideas. We have performed these tests under most severe conditions in order to determine the characteristics and fundamental of GLR.

Our basic conclusions are that GLR is an extremely promising method for the detection of abrupt changes in dynamic systems. The method has been successfully applied in several applications [7-14], it is amenable to detailed performance analysis, and it offers a range of implementations of varying levels of complexity. Failure mode distinguishability and parameter sensitivity do cause problems, as one might expect, but we have seen that one can analyze these issues in great detail. Specifically, the information matrix allows one to determine the absolute limits on one's ability to distinguish among several failure modes. In addition, we have seen that detector response in the presence of parameter errors differs markedly from the consistent response due to a failure.

Issues that should be examined in the future include:

(A) Failure Mode Distinguishability

    (1) The orthogonalization of failure modes in order to improve distinguishability should be considered. This includes the

possibility of a dual-mode procedure, in which one first

detects failures with a "universal" signature and then

uses orthogonal signatures to determine the actual failure

mode.

(2) The implementation and study of the performance of CGLR.

(3) The study of the correlation behavior of the $\ell_i(k,\theta)$

under various failure hypotheses in order to determine

"smart" detection rules that make decisions based on

patterns of likelihood ratios.

(4) The study of other failure modes (hard-over, added noise,

etc.). Can we find useful signatures (perhaps the same

as for the basic four modes) to use in detecting these

failures? What about combinations of failure modes

(e.g., bias + hard-over, noise + bias, multiple failures)?

GLR is general enough to deal with all of these. Our

aim here is to define some useful groundrules for

choosing an appropriate set of failure signatures

(perhaps modulated by $\hat{x}$ or u, as in the hard-over cases).

(B) GLR Robustness -- The results obtained so far indicate that parameter

uncertainties can lead to difficulties in correctly detecting system

failures. However, our results also indicate that such parameter uncer-

tainties lead to very distinctive likelihood ratio patterns. Several

issues must be considered.

(1) The statistics of the residuals in the presence of
     parameter errors are somewhat complex but should be
     examined.

(2) Parameter errors lead to residual signatures much like
     failures. These should be examined in order to under-
     stand how they can confuse GLR and how to avoid such
     confusion.

(3) As mentioned in the preceding section, the likelihood
     ratio patterns when parameter errors are present have
     very distinctive forms. Thus we are interested in
     determining smart detection rules. One appealing as-
     pect of smart rules is the use of provisional decisions.
     That is, based on initial information we may make a
     decision to pull out a particular instrument or activate
     a back-up. We can then continue to monitor residuals
     to obtain corroboratory information concerning the failure
     (see [12-14] for an approach along these lines in a dual-
     redundant system). In this way we avoid catastrophic
     delays when a large failure really is there.

(4) It is felt that CGLR and SGLR will be less sensitive to
     parameter errors. This should be examined via simulation.

(5) In many problems there are system nonlinearities, as well
     as parameter errors, that must be dealt with. One should
     examine modifications of GLR that can deal with this.

For example, one can develop Extended GLR (EGLR),

the analog of the extended Kalman filter.

(6) Another important area that can be studied is the ef-

fect of discrepancies in system state dimension. For

example, if a GLR system is based on the 2-dimensional

F-8 model, how well will it behave on a real F-8,

which is much more complex?

(C) <u>Complexity and Performance</u> -- All of the results up to this point have

been aimed at understanding the qualitative features of GLR and in developing

a set of analytical tools. This insight and these techniques are essential

to the development of a GLR design methodology. This methodology clearly

must come to grips with a number of tradeoff issues:

(1) We need to develop several measures of performance, such

as delay time in detection, false alarm rate, etc. Such

measures may be of more importance in some problems than

in others -- e.g. false alarms may be more tolerable in

highly redundant systems. Also, one must develop methods

of evaluating such measures. We have done that in some

cases, but if we develop detection rules dealing with

patterns of likelihood ratios, we will have to develop

new analytical techniques.

(2) In many cases -- e.g. when several identical sensors are

available -- one can use much simpler detection systems

than GLR, such as voting. In addition, redundancy can

greatly reduce distinguishability and sensitivity

problems. For example, GLR detectors looking for

failures in identical instruments would register

similar responses to parameter errors, and this si-

milarity could be exploited to avoid declaring a

failure. The tradeoffs involved in determining what

redundant design is most appropriate is a problem

worthy of further study.

Our research efforts have been extremely worthwhile in developing GLR

and in extracting the key issues involved in implementing a GLR- based

detection system. This method has been successfully applied in several

applications, and it is our feeling that the GLR approach has the potential

to be a useful design tool in future aerospace applications.

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